

An Empirical Analysis of the Queuing Theory and its Application to Customer Satisfaction in a Small and Medium Enterprises (SMEs): A Study of Danjalele Enterprise, Funtua, Kastina State

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Abstract: Customers are often forced to wait in line whenever the service facility is busy. However, too much waiting in line causes inconvenience to customers and economic costs to firms. Queuing theory is a mathematical approach for analysing waiting lines (queues). The effect of queuing in relation to the time spent by customers to access services is increasingly becoming a major source of concern. Moreover, For the reason that keeping customers waiting too long involves excessive cost to them (waiting cost). Providing too much service capacity to operate a system involves excessive cost. But not providing enough service capacity results in excessive causes the waiting line to become excessively long. The ultimate goal is to provide an economic balance between the cost of service and the cost associated with the waiting for that service. The objective of this paper is to examine the queuing congestion in Danjalele Enterprise, Funtua, and develop a suitable queuing model for the system. The study concludes that an alternate model may be more efficient when compared with the existing one. The paper recommends that the management of the Enterprise should adopt a multi-server model as this will further attract potential customers to patronize the products and thereby increasing their satisfaction.

Keywords: Queue; M/M/1 model; FIFO; Service cost; Utilization factor; Waiting cost; Waiting time, optimization.

1. INTRODUCTION

Queuing theory was originated by a Danish engineer named A.K. Erlang. In 1909 Erlang experimented with fluctuating demand in telephone traffic. Eight years' later A.K. Erlang published a report addressing the delays in automatic dialing equipment. At the end of World War II, Erlang's early work was extended to more general problems and to business applications of waiting lines.

Queue (waiting lines) seems to be part of our everyday life, for example, at the hospital, filling station, banks, bus stop, supermarkets, restaurant and etc. Kandemir-Cavas and Cavas, (2007) opined that queues form when the demand for a service exceeds its supply. According to Obamiro, (2010) many customers or patients, waiting in lines or queuing is annoying as it causes not only inconvenience, but also frustration to people's daily lives. Often, customers may be

discouraged from pursuing valuable services by a sheer length of the waiting line. At other times, waiting might even cause the customer to delay or miss important events.

Davis, Aquilano and Chase, (2003) observed that the way and manner in which managers address the waiting line issue is critical to the long term success of their firms. He added that, unmanaged queues are detrimental to the gainful operation of service systems and results in a lot of other managerial problems. However, waiting time difficulties can be managed by using queuing model to determine the waiting line performance such as: average arrival rate of customers, average service rate, system utilization factor, cost of service and the probability of a specific number of customers in the system (Okoye, 2011). In order to reduce the time spent in waiting systems, one solution would be to supplement the checkout clerks (Nafees A, 2007). The purpose of this study is to provide insight into the general background of queuing theory and its associated terminology, and how queuing theory can be used to model small and medium scale businesses.

2. OBJECTIVES OF THE STUDY

The objectives of this paper include the following: -

- To study the congestion in the check-out counter of the Danjelele Enterprise.
- To validate the data collected in the investigation.
- To develop a queuing model for the study.
- To propose an alternate model and compare its efficiency levels with the existing model.

3. LITERATURE REVIEW

There are many studies conducted on done in queuing theory. However, the few studies selected are reviewed below.

Jhala and Bhathawala (2016) studied applications of queuing theory in banking sector. In this study, multiple servers model was employed in order to reduce the traffic congestion. The study adopted primary data using observation method. The Waiting and service Costs were determined with a view to determining the optimal service level. The results of the analysis showed that average queue length, waiting time of customers were analyzed for simultaneous efficiency in customer satisfaction and cost minimization through the use of a Multi-server queuing Model. The study recommended that multiple servers model should be maintained in order to reduce the traffic congestion.

Munirat, Nwaiwu and Aina (2015) examined the Impact of Queuing Theory on Customer Satisfaction: A Review of Performance, Trends and Application in Banking Practice. The purpose of this paper is to explain how queuing theory can be used to model small and medium scale businesses. The study adopts regression analysis and model with the help of SPSS in analyzing the data collected through primary and secondary means. The study concludes that most customers in banks are not satisfy base on the queue they experience before been attended to. The study recommends the need to provide an effective and efficient application of queuing theory which can be of particular benefit in banks with high-volume out-customer workloads and/or those that provide multiple points of service. By better understanding queuing theory, service managers can make decisions that increase the satisfaction of all relevant groups-customers, employees, and management.

Bonga (2014), studied Queuing Theory and its application on Customer Satisfaction in Small and Medium Enterprises. The study evaluates the effectiveness of a queuing model in identifying the restaurant queuing system efficiency parameters. It uses Little's Theorem to analyze data collected from Croc Foods Restaurant in Beitbridge over a three-week period. The study revealed that on average 60 customers arrive every hour and the service rate is 84 customers per hour. Using the Queuing Theory calculator, the system utilization factor was 71.43%, the probability of zero customers waiting 0.2857, average number of customers waiting 1.782 and average waiting time 0.0298 hours. The study compared the single server model against multi-server model and concluded that M/M/1 model was not the best for Croc Foods restaurant. Using a questionnaire of 171 respondents, the study found out that about 43.3% of customers are not satisfied about the nature of waiting lines and about 69% customers have at least turned away at regular occasions due to the queues. The study recommended that effective application of the model can help to improve access to quality services which is viewed as key to decrease the incidence of customer base and loyalty and hence profitability.

Bakari, Chamalwa and Baba (2014) conducted a study on the application of Queuing Process to Customer Service Delivery using Fidelity Bank Plc, Maiduguri branch. The study uses primary data collected through observation, in which the number of customers arriving at the ATM service point of Fidelity Bank Plc located in West-End, Maiduguri was recorded, as well as each customer's arrival and service time respectively. The period for the data collection was during busy working hours (i.e. 8:00am to 4:00pm) and for a period of ten (10) working days. The results find that the traffic intensity (ρ) is 0.96. Since the obtained value of the traffic intensity, otherwise known as the utilization factor was less than one (i.e. $\rho < 1$), they concluded that the system operates under steady-state condition. Thus, the value of the traffic intensity, which is the probability that the system is busy, implies that 95% of the time period considered during data collection the system was busy as against 4% idle time. This indicates high utilization of the system.

Brahma, (2013) studied the Application of Queuing Theory and Customer Satisfaction: A Review of Terminology, Trends, And Applications to Hospital Practice. The aim of this paper was to give the reader a general understanding of concepts, current technology, and applications of queuing theory as it relates to patient satisfaction and waiting time. The study found out that out of 2,500 outpatients the doctors are servicing over 1,000 patients daily. The study recommends the need for the management of the hospitals to Utilize Automated queuing technology (AQT) because it can easily track variables such as patient arrival and departure time; patterns of arrival, prescription fill time, waiting time, and individual staff member productivity. In addition, Automated queuing technology can track numerous points of service and different service categories (ie, certain patients may get priority service or can be used to track patient counseling) if desired. Finally, Automated queuing technology can also provide hospital patients with information that can directly improve their queuing experience, such as with a ticket with a unique number and the estimated wait time. This makes for a less confusing, more relaxed, and much more positive waiting environment for the patient.

KAMAU, (2012) examined the impact of waiting lines management and customer satisfaction in Commercial Banks in Kenya. The main aim of this study was to find out how waiting line management influences customer satisfaction in Kenyan banks. To achieve the main objective, the study determined the effect of waiting lines management on customer satisfaction, how commercial banks manage waiting lines and the challenges faced by commercial banks while managing waiting lines. The population study consists of all the 43 registered commercial banks in Kenya. The study adopts primary data using self-administered questionnaire and the analysis was done using descriptive statistics generated from Statistical Package for Social Sciences (SPSS). The study concludes that majority of commercial banks customers in Kenya are not satisfied with management of waiting. The main area where customers are dissatisfied is the perceived waiting time. Besides, bank customers are not satisfied with information provided in the waiting room and the waiting environment within the banks. Customers become less satisfied due to perceived longer waiting times that they have to spend queuing at the bank. Customers are not satisfied with management of waiting lines despite the fact that banks have implemented lines management measures such as use of physical barriers, automatic queue measurement systems, seamless banking, internet banking, mobile banking and agency banking. The study recommends that commercial banks in Kenya should invest in the provision of appropriate information to the customers and commercial banks in Kenya should increase customer satisfaction by adopting strategies such as use of signage and signaling systems, use of interactive communication and advertisement on the television screens and allowing visitors' freedom of movement while keeping them informed about the expected waiting time or giving them a guaranteed reception time. The study also recommends further study in efficiency of line management technologies in Kenya banking sector.

Tom and Lucey (2010) conducted a Field Study on the Effect of Waiting Time on Customer Satisfaction. A field study was conducted in two different supermarkets during busy and slow hours of operation and with slow and fast checkers to determine the effects of objective waiting time, perceived waiting time, and serve time on customer satisfaction with the server and the store. The two-way analyses of variance (ANOVAs) indicated that the store environment (busy or slow) and the speed of the checkers (faster or slower) did affect objective waiting time, perceived waiting time, and serve time., there were also store-specific findings. For example, in one of the supermarkets, customers reported higher satisfaction with slower checkers than faster checkers, suggesting that longer waiting times sometimes result in more satisfied customers than shorter waiting times.

Nafees, (2007) conducted an investigation on Queuing Theory and its Application: Analysis of the Sales Checkout Operation in Ica Supermarket. The objectives of this paper is to examine the efficiency of the models in terms of utilization and waiting length, hence increasing the number of queues so customers will not have to wait longer when

servers are too busy. The study uses queuing simulation for a multiple server models to obtain a sample performance result. This study requires an empirical data which may include the variables like, arrival time in the queue of checkout operating unit (server), departure time, service time, etc. A questionnaire is developed to collect the data for such variables and the reaction of the ICA Supermarket from the customers separately.

The model contains five servers which are checkout sales counters; attached to each server is a queue. The study revealed that with the increasing number of customers coming to ICA for shopping, either for usual grocery or for some house wares, there is a trained employee serving at each service unit.

4. METHODOLOGY

1) Data Collection

The data was collected for the check-out counter in Sule Mamuda Enterprise for a period of 3 weeks. A stop-watch in the process. The data collection was done for three different timings during the working hours of the store (enterprise): morning, afternoon and evening. One week were allotted to each timing.

2) Data Validation

The inter-arrival times and the service times were found to follow Poisson and exponential followed distribution respectively. The number of servers was one as there was only a single check-out in the store. Thus, it was found that the system was fit for the m/m/I queuing model.

3) Development of Queuing Model

Certain assumptions that were made during the research are:

- Queue discipline was assumed to be first-come-first-serve (FCFS) type.
- Reneging, balking and jockeying of the customers were not taken into consideration in the study.
- The population source was taken to be infinity.
- Infinite number of customers are allowed in the system.

According to Kendall's notation, the model for the system could be represented as m/m/i: FCFS/ ∞/∞ . The following parameters were analysed for the model:

Mean customers arrival rate = λ

Mean service rate = c

Utilization factor, $P = \frac{\lambda}{\mu}$

Probability of zero customers in the system $P_0 = 1 - P$

Probability of having n customers, $P_n = P_0 P^n = (1 - P)P^n$

Average number of customers in the system, $L_s = \frac{\lambda}{\mu - \lambda}$

Average number of customers in the queue, $L_q = L_s - \frac{\lambda}{\mu}$

Average time spent in the system, $W_s = \frac{1}{\mu - \lambda}$

Average time spent in the queue, $W_q = \frac{P}{\mu - \lambda}$

Moreover, the model parameters were analysed for each of three timings individually and a comparison was made amongst the three timings. Finally, an alternate model was proposed and the efficiency levels were compared to that of the existing model.

5. RESULTS AND DISCUSSION

The results found from the analysis are as follows:

A. Analysis of the parameters of the queuing model for the whole system

Number of servers = 1

Mean customers arrival rate = $\lambda = 16$ customer

Per hour, mean service rate $\mu = 23$ customer per hour

Utilization factor = $P = \frac{\lambda}{\mu} = 0.6957$

Probability of zero customers in the system $P(0) = 1 - P = 0.3043$

Average number of customers in the system.

$L_s = \frac{\lambda}{\mu - \lambda} = 2.2857$

Average number of customers in the queue

$L_q = L_s - \frac{\lambda}{\mu} = 1.59$

Average time spent in the system

$W_s = \frac{1}{\mu - \lambda} = 0.1429$ hours

Average time spent in the queue

$W_q = \frac{P}{\mu - \lambda} = 0.0994$ hours

Table 1: Results of the m/m/i: FCFS/ ∞/∞ Model for the whole System

N	Probability	Cumulative Probability, P(0)
0	0.3043	0.3043
1	0.2117	0.5160
2	0.1473	0.6633
3	0.1025	0.7658
4	0.0713	0.8371
5	0.0496	0.8867
6	0.0345	0.9212
7	0.0240	0.9452
8	0.0167	0.9619
9	0.0116	0.9735
10	0.0081	0.9816
11	0.0056	0.9872
12	0.0039	0.9911
13	0.0027	0.9938
14	0.0017	0.9955

From table 1: The result indicates that the probability of customers in the queue decreases with increasing number of customers. The probability of having one customer in the queue is 0.2117 and the probability of having three customers is 0.1025. the cumulative probability is quickly approaching. For example, for 14 customers it is 0.0017. This implies that it is rare to have more than 14 customers in the queue under normal conditions.

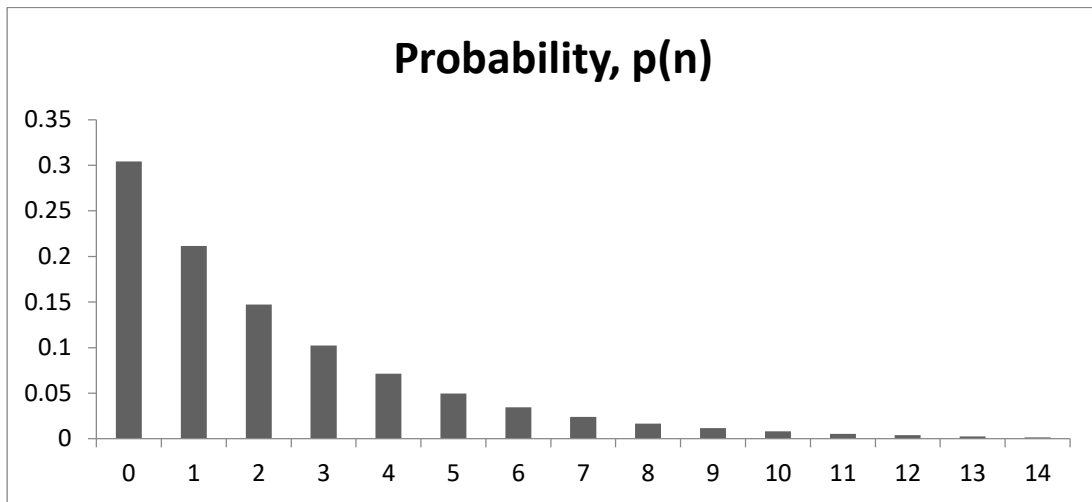


Fig 1. Plot of P(n) Vs N for the whole system

B. Analysis of the parameter of the queuing model for each of the 3 timings individually:

i) In the evening

Number of servers = 1

Mean customers arrival rate, $\lambda = 18$ customers per hour.

Mean service rate, $\mu = 25$ customers per hour

Utilisation factor, $P = \frac{\lambda}{\mu} = 0.72$

Probability of zero customers in the system $P(0) = 1 - P = 0.28$

Average number of customers in the system

$$L_s = \frac{\lambda}{\mu - \lambda} = 2.5714$$

Average number of customers in the queue

$$L_q = L_s - \frac{\lambda}{\mu} = 1.8514$$

Average time spent in the system

$$W_s = \frac{1}{\mu - \lambda} = 0.1429 \text{ hours}$$

Average time spent in the queue

$$W_q = \frac{P}{\mu - \lambda} = 0.1029 \text{ hours}$$

Table 2: Results of the m/m/i: FCFS/∞/∞ Model for the Evening Congestion

N	Probability	Cumulative Probability, P(n)
0	0.28	0.28
1	0.2016	0.4816
2	0.1452	0.6268
3	0.1045	0.7313
4	0.0752	0.8065
5	0.0542	0.8607
6	0.0390	0.8997
7	0.0281	0.9278
8	0.0202	0.9480

9	0.0146	0.9626
10	0.0105	0.9731
11	0.0075	0.9806
12	0.0054	0.9860
13	0.0039	0.9899
14	0.0028	0.9927
15	0.0020	0.9947
16	0.0015	0.9962
17	0.0011	0.9973

The results from table-2 also shows that the probability of customers in the queue decrease with increasing number of customers. The probability of having one customer in the queue is 0.0216 and the probability of having three customers is 0.1045. The cumulative probability is quickly approaching 1, for example for 17 customers it is 0.0011. This implies that it is very uncommon to have more than 17 customers in the queue under normal conditions.

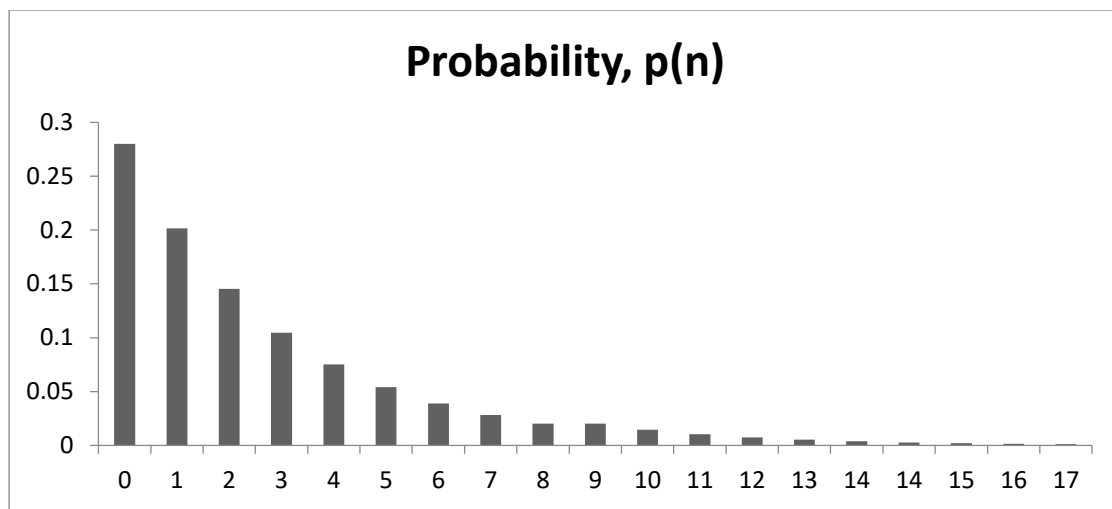


Fig 2.

ii) In the morning

Number of servers = 1

Mean customers arrival rate, $\lambda = 14$ customers per hour.

Mean service rate, $\mu = 22$ customers per hour

Utilisation factor, $P = \frac{\lambda}{\mu} = 0.6364$

Probability of zero customers in the system $P(0) = 1 - P = 0.3636$

Average number of customers in the system

$$L_s = \frac{\lambda}{\mu - \lambda} = 1.75$$

Average number of customers in the queue

$$L_q = L_s - \frac{\lambda}{\mu} = 1.1136$$

Average time spent in the system

$$W_s = \frac{1}{\mu - \lambda} = 0.125 \text{ hours}$$

Average time spent in the queue

$$W_q = \frac{P}{\mu - \lambda} = 0.0796 \text{ hours}$$

Table 3: Results of the m/m/i: FCFS/∞/∞ Model for the Morning Congestion

N	Probability	Cumulative Probability, P(n)
0	0.3636	0.3636
1	0.2314	0.5950
2	0.1473	0.7423
3	0.0937	0.8360
4	0.0596	0.8956
5	0.0380	0.9336
6	0.0242	0.9578
7	0.0154	0.9732
8	0.0098	0.9830
9	0.0062	0.9892
10	0.0040	0.9932
11	0.0025	0.9957
12	0.0016	0.9973

From table 3, it is evident that the probability of customers in the queue decreases with increasing number of customers. The probability of having one customer in the queue is 0.2314 and that of having three customers is 0.0937. The cumulative probability is quickly approaching 1. For example, for 12 customers is 0.0016. This implies that it is rare to have more than 12 customers in the queue under normal conditions.

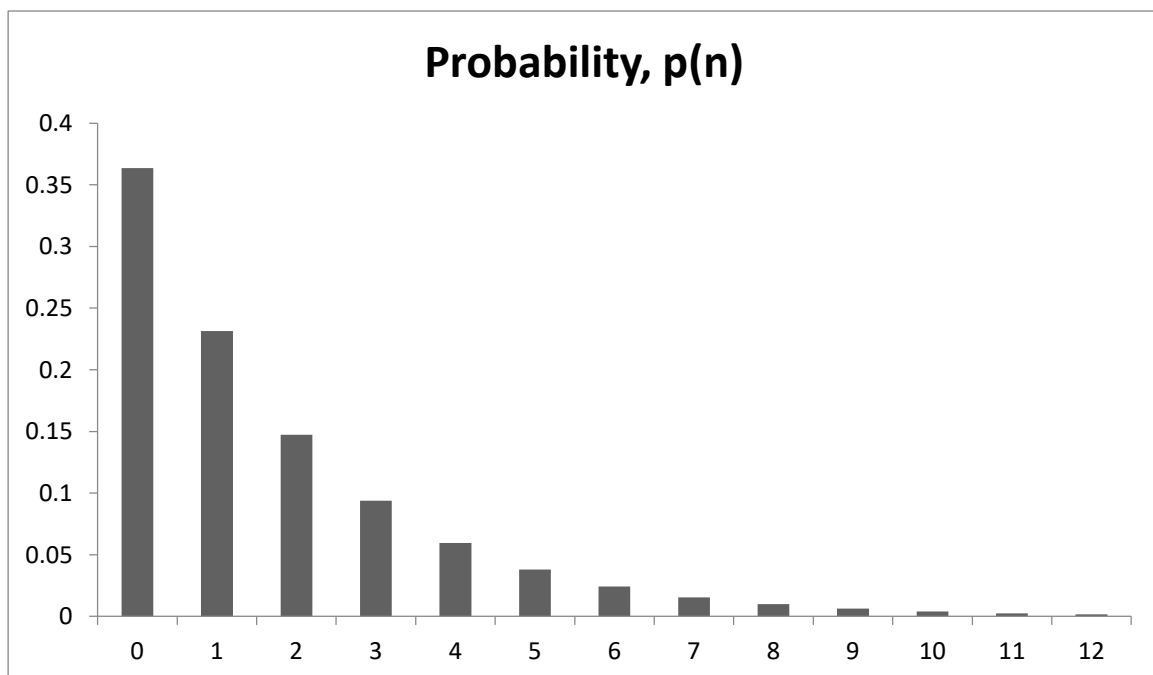


Fig 3

iii) In the afternoon

Number of servers = 1

Mean customers arrival rate, $\lambda = 13$ customers per hour.

Mean service rate, $\mu = 21$ customers per hour

Utilization factor, $P = \frac{\lambda}{\mu} = 0.6190$

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Probability of zero customers in the system $P(0) = 1 - P = 0.381$

Average number of customers in the system

$$L_s = \frac{\lambda}{\mu - \lambda} = 1.625$$

Average number of customers in the queue

$$L_q = L_s - \frac{\lambda}{\mu} = 1.006$$

Average time spent in the system, $W_s = \frac{1}{\mu - \lambda} = 0.125$ hours

Average time spent in the queue, $W_q = \frac{P}{\mu - \lambda} = 0.0774$ hours

Table 4: Results of the m/m/i: FCFS/∞/∞ Model for the Afternoon Congestion

N	Probability P(n)	Cumulative Probability, P(n)
0	0.3810	0.3810
1	0.2358	0.6168
2	0.1460	0.7628
3	0.0904	0.8532
4	0.0559	0.9091
5	0.0346	0.9437
6	0.0214	0.9651
7	0.0133	0.9784
8	0.0082	0.9866
9	0.0051	0.9917
10	0.0031	0.9948
11	0.0019	0.9967
12	0.0012	0.9979

From table 4, it is again seen that the probability of customers in the queue decrease with increasing number of customers. The probability of having one customer in the queue is 0.2358 and the probability of having three customers is 0.0904. The cumulative probability is quickly approaching 1. For example, for 12 customers it is 0.0012, implying that it is rare to have more than 12 customers in the queue under normal conditions.

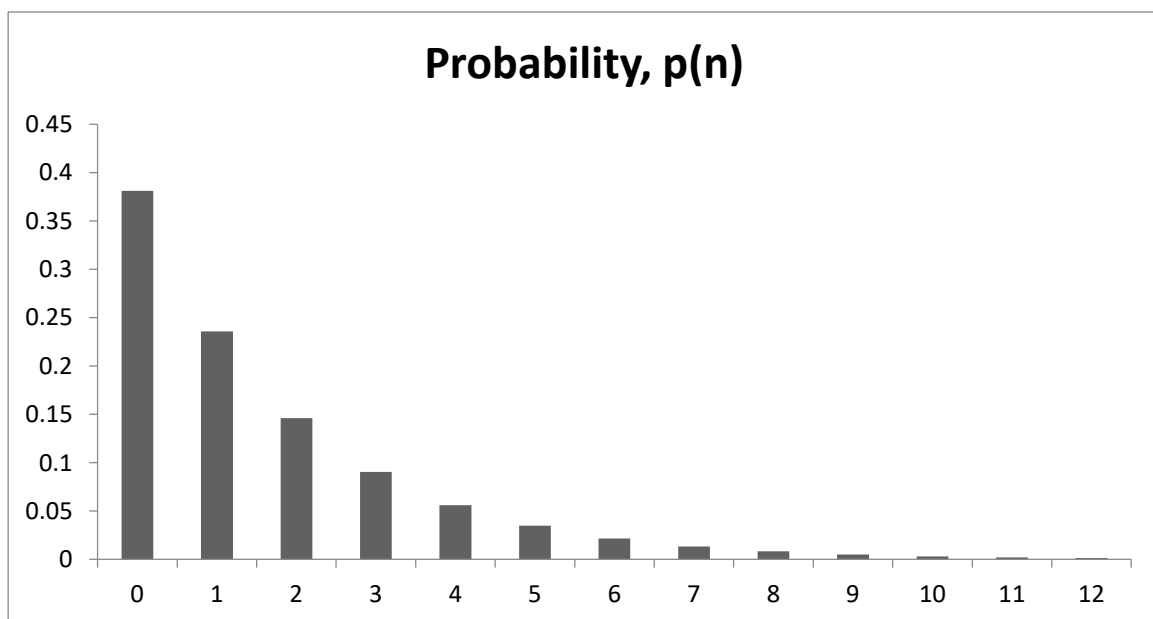


Fig.4: Plot of P(n) Vs N for the afternoon time

TABLE 5: Comparison of the Parameters of the M/M/I Model for the Three Timings

Parameters	Evening	Morning	Afternoon
Λ	18	14	13
μ	25	22	21
P	0.72	0.6364	0.6190
P(0)	0.28	0.3636	0.3810
Ls	2.5714	1.75	1.625
Lq	1.8514	1.1136	1.006
Ws	0.1429	0.125	0.125
Wq	0.1029	0.0796	0.0774

From table 5: the result also shows the efficiency under three different timing. The traffic intensity significantly changed from 72% to 63.64% and to 61.9%. This shows that the nearer the traffic intensity to zero the more efficient the queue system which have the potential of attracting more customers.

C) Comparison of the Single Server Model with A Proposed Multi-Server Model

The Danjelele Enterprise selected for this study used the single server model $m/m/1: FCFS/\infty/\infty$. But for efficiency purposes, it is worth to assume a multi-server model being used by the same store (enterprise) so as to compare the efficiency levels. To compare, the study has proposed a multi-server model ($m/m/2$). The efficiency parameters for the two models are shown in Table 6 below:

Table 6: Comparison of the Queuing Models

PARAMETERS	M/M/1	M/M/2
Λ	16	16
μ	23	23
P	0.6957	0.3478
P(0)	0.3043	0.4839
Ls	2.2857	0.7914
Lq	1.59	0.0957
Ws	0.1429	0.0495
Wq	0.0994	0.006

Table 6 above shows the efficiency parameters under two different queuing models. The traffic intensity has changed from 69.57% to 34.78%. This shows that as more servers (check-out counters) are introduced, then the store becomes less busy. This may attract more customers.

6. CONCLUSION AND RECOMMENDATIONS

In the case of the Danjelele Enterprise selected as a case study, the results throw light on some important issues about the operation mode of the waiting line. The customers have to wait an average of 0.1429 hours in the system. The average number of customers who have wait is 1.59 and 69.57% of arriving customers have to wait to be served. These results show that there is need to improve the operations that occur within the waiting line. If the store continues to use the single server waiting line, the number of waiting customers will increase. In general, there is need to improve the service rate.

Based on the conclusion, the paper recommended that the store should introduce another check-out counter in order to lower the congestion and also to attract more customers. However, the decision for introduction of another check-out counter must also focus on cost consideration (a break-even analysis).

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